UNIT 1: REAL NUMBERS AND INTERVALS

Exercise 1: Classify the following numbers and place them on the number line

a) π ; $7/5$; $0.477777\cdots$; $-\sqrt{9}$; -2 ; $\sqrt{3}$; $\sqrt{-25}$; $14/7$; $\sqrt[3]{-125}$

b) ϕ ; $20/7$; $\sqrt[4]{-16}$; $7.22222\cdots$; $-\sqrt{16}$; -5 ; $\sqrt{2}$; $21/3$; $-\sqrt[3]{-27}$

Exercise 2: Turn the following decimal numbers into fractions

a) $4.37289 =$

b) $52.\overline{934} =$

c) $7.29\overline{5} =$

d) $2.12131415\cdots$

e) $7.48\overline{295} =$

f) $5.121212 =$

g) $23.\overline{73} =$

h) $0.\overline{9} =$

Exercise 3: Decide if the following affirmations are true or false and why

a) All integer numbers are natural numbers.

b) Rational numbers are not real numbers.

c) The product of two rational numbers can never be a natural number.

d) The difference of two irrational numbers is always an irrational number.

Exercise 4: Decide if the following affirmations are true or false and why

a) The quotient of two integer numbers can never be a natural number.

b) Irrational numbers are rational numbers.

c) The sum of two irrational numbers is always an irrational number.

d) Not a single real number is an irrational number.

Exercise 5: Sam does an experiment to find how long it takes an apple to drop 2 m. The theoretical value is 0.64 s, but Sam measures 0.62 s. Find the percentage error.

Exercise 6: People say that a few summers ago we reached 45ºC in Córdoba, but actually the highest temperature was only 43.8 ºC. Find the percentage error.

Exercise 7: I've used Thales' theorem on a sunny day to measure the height of Mount Everest, and I got 8823m, and the Giralda in Seville, and I got 73.5m. Then I checked the Wikipedia and found out that the real measures are 8848m and 98.5m. Work out the value of the absolute, relative and percentage errors and explain the results.

Exercise 8: Find the absolute and relative errors when we approximate $\sqrt{2}$ by $17/12$

Exercise 9: Archimedes used $22/7$ as an approximation to π. If I have a circle with a radius of 25 m, what's the percentage error when using that approximation to find its area?

Exercise 10: Round and truncate the number $e \simeq 2.7182818285$ to three significant figures and estimate both the absolute and relative errors. Which approximation is better? Why?

Exercise 11: When they ask me I always say that my height is 1.55 m, but the truth is that I am only 153 cm high. Find the absolute, relative and percentage errors that I commit each time that I lie like that.

Exercise 12: Work out:

a) $\left(\dfrac{5}{4}\right)^{-1} - \dfrac{2}{4}\cdot\left(\sqrt{\dfrac{1}{9}} - \dfrac{7}{5}\cdot\dfrac{3}{7} + 1\right) =$

b) $\left(1 - \dfrac{2}{3}\right)^{-2} - \left(\dfrac{1}{5} - \dfrac{3}{2}\right)^{-1} =$

c) $\dfrac{10}{3}\cdot\left(\dfrac{7}{4} - \dfrac{1}{2}\right)^{-2} - \left(3 - \dfrac{1}{2}\right)^{-1} =$

d) $\dfrac{3/2}{5/3} - \left(\dfrac{1}{4}:\dfrac{2}{7}\right)^{-2} + \left(\sqrt{\dfrac{2}{5}\cdot\dfrac{18}{5}}\right)^{-1} =$

Exercise 13: A regular season of Game of Thrones has ten episodes. And I am gonna watch them all in a single day. In the morning I've watched two fifths of the episodes, and in the afternoon, two thirds of the remaining ones. How many eps do I still have to watch tonight?

Exercise 14: Yesterday I went shopping and I spent one third of my money in a phone store, and then two fifths of what I had left in a stationery store. If I still have 5€, how much money did I have in the beginning?

Exercise 15: Dany ate one quarter of a cake and then her nephew Jon ate two sevenths of what was left. If there are still 250 g of cake remaining, what was the weight of the cake?

Exercise 16: We are going on a field trip. On the first stop I "lose" one fifth of the students, and on the second stop I "lose" two thirds of the remaining students. If even after that I still "have" thirty-two students, how many people were we when we left Córdoba this morning?

Exercise 17: I've planted a fifth of a field with potatoes, two thirds of the remaining with tomatoes, three fifths of the remaining with lettuces, and in the 448 m^2 left I am planning on planting carrots. What's the total surface of my field?

Exercise 18: Write the following intervals as inequalities and represent them on the number line:

a) $[-9, +\infty)$ b) $(-1, 5]$ c) $(-2, -1)$

d) $[-1, 8]$ e) $(-\infty, 3]$ f) $(-\infty, +\infty)$

Exercise 19: Write the following inequalities in interval notation and then represent them on the number line.

a) $3 \leq x < 8$ b) $-4 < x \leq 3$ c) $-5 \leq x$

d) $-3 < x < -6$ e) $x < 7$ f) $2 \leq x \leq 3$

Exercise 20: Write as an interval:

a) $(-5, 4) \cup [-2, 0]$ b) $(-9, -4) \cup [-4, 1]$ c) $[-4, 1) \cap (-1, 2]$

d) $(-7, -2) \cup (-2, 1]$ e) $[-2, 2] \cap [2, 3]$ f) $[-5, 4) \cap (-1, 1]$

Exercise 21: Study these unions and intersections of intervals:

a) $(-2, 3) \cap (1, 4)$ b) $(-5, 7) \cup [3, 6)$ c) $(-2, 5] \cap [5, 8]$

Exercise 22: Represent $\sqrt{34}$ on the number line using just a compass and a ruler

Exercise 23: Represent $\sqrt{41}$ on the number line using just a compass and a ruler